Finite temperature QCD in the quark-composites approach

F. Palumbo [*]
INFN – Laboratori Nazionali di Frascati
P. O. Box 13, I-00044 Frascati, ITALIA
Internet: `palumbof@lnf.infn.it`

We investigate QCD at finite temperature in the quark composites approach, which is based on the use of quark composites with hadronic quantum numbers as fundamental variables. We find that chiral symmetry restoration and quark deconfinement are one and the same first order phase transition, whose critical temperature, in a one loop approximation, is $T = 2\sqrt{\Omega}\rho^{-2}m_\pi$, where m_π is the pion mass, $\Omega = 24$ the number of up and down quark components, and ρ a parameter of order 1 whose precise value can be determined by the study of the pion-pion interaction.

[*]This work has been partially supported by EEC under TMR contract ERB FMRX-CT96-0045

The idea that a spontaneously broken symmetry can be restored at sufficiently high temperature [1] has found a natural application in QCD. Since quantitative investigations of this problem rely so far on numerical simulations, it is not easy to establish whether the chiral symmetry restoration is a first or second order phase transition, a feature which according to many authors depends on the number of flavors [2]. The situation is somewhat complicated by the possible interplay between the restoration of chiral symmetry and quark deconfinement, which in a recent paper have been found to coincide [3].

We study QCD at finite temperature in the quark composites approach [4] with two flavors. We find that chiral symmetry restoration and quark deconfinement are indeed one and the same first order phase transition. Although the validity of our resul is restricted to two flavors, the reason why the transition is first order does not seem to depend on the number of flavors. We find the critical temperature

$$T = \frac{2\sqrt{\Omega}}{\rho^2}\, m_\pi. \qquad (.1)$$

In the above equation m_π is the pion mass, $\Omega = 24$ is the number of up and down quark components, and ρ is a parameter of order 1 whose precise value can be determined by the study of the pion-pion interaction.

While the conclusion about the nature of the phase transition is unavoidable in the present context, we cannot say anything about the accuracy of the value of the critical temperature without an estimate of its corrections. Here it could help a numerical simulation done using the action of the quark composites approach. Indeed this approach is based on the idea that if a significant part of the binding mechanism of hadrons is accounted for by the use of quark composites with hadronic quantum numbers as fundamental variables, the residual interaction can be sufficiently weak for a perturbative treatment. If this is true the present formulation should also make it easier the numerical evaluation of the quark determinant.

The idea at the basis of the quark composites approach is realized by assuming the composites with the quantum numbers of the nucleons as new integration variables in the Berezin integral which defines the partition function [5], and replacing the composites with the quantum numbers of the chiral mesons by auxiliary fields [6]. Then irrelevant operators which provide the kinetic terms for the composites are added to the standard action, and the quark action is treated as a perturbation. The expansion parameter cannot obviously be the gauge coupling constant, but it is instead a dimensionless constant appearing in the definition of the composites. In conclusion the perturbative expansion is formulated in terms of phenomenological fields, whose effective coupling constants, however, are given to any order by integrals, depending on the order, over the gluon and quark fields. The gluon fields can be treated, according to the dynamical regime, perturbatively or non perturbatively wr to the gauge coupling. It is an intrinsic, fundamental feature of the approach that the quarks, as a consequence of the spontaneous chiral symmetry breaking are nonpropagating particles which appear only in virtual states: In other words their confinement is built in already at the perturbative level, unlike the standard perturbation theory where the quarks appear as physical particles whose confinement is intrinsically nonperturbative.

The framework of our approach appears most convenient for the study of the phase transitions associated to chiral symmetry restoration and quark deconfinement. In this paper we investigate QCD at finite temperature and zero barion density, a purpose for which it is sufficient to introduce only the chiral composites. We adopt a regularization on a euclidean lattice because it seems natural dealing with composites, and it allows us to handle also the nonperturbative regime of the gluons. We assume the modified partition function

$$Z = \int [dU] \int [d\overline{\lambda} d\lambda] \exp[-S_{YM} - S_C - S_Q], \qquad (.2)$$

where S_{YM} is the Yang-Mills action, S_Q is the action of the quark fields and S_C is an irrelevant operator which provides the kinetic terms for the quark composites with the quantum numbers of the chiral mesons. $\lambda^a_{\tau\alpha}$ is the quark field with color, isospin and Dirac indices a, τ and α respectively, and the gluon field is associated to the link variables U_μ.

All the elementary fields live in an euclidean lattice of spacing a, whose sites are identified by fourvectors x of spatial components $x_k = 0, ...N$ and time component $t = 0, ...N_t$, and satisfy periodic boundary conditions, with the exception of the quark fields which are antiperiodic in time: $\lambda(x) = \lambda(x + Ne_k) = -\lambda(x + N_t e_t)$, e_k and e_t being the unit vectors in the k and time directions. The temperature is $T = (N_t a)^{-1}$.

The quark action, with the notation

$$(f, g) = a^4 \sum_x f(x)g(x), \qquad (.3)$$

can be written

$$S_Q = (\overline{\lambda}, Q\lambda). \qquad (.4)$$

The quark wave operator is

$$Q_{x,y} = m_Q \delta_{xy} + \frac{1}{2a} \sum_\mu \gamma_\mu U_\mu(x) \delta_{x+e_\mu,y}. \qquad (.5)$$

We adopt the standard conventions

$$\begin{aligned} &\mu \in \{-4, \ldots, 4\}, \quad e_{-\mu} = -e_\mu, \\ &\gamma_{-\mu} = -\gamma_\mu, \\ &U_{-\mu}(x) = U^+_\mu(x - e_\mu). \end{aligned} \qquad (.6)$$

and omit the Wilson term for a reason to be explained later.

The chiral composites are the pions and the sigma

$$\pi_h = i\, k_\pi\, a^2\, \overline{\lambda}\gamma_5\tau_h\lambda, \quad \sigma = k_\pi\, a^2\, \overline{\lambda}\lambda. \tag{.7}$$

γ_5 is assumed hermitean and the τ_k's are the Pauli matrices. An arbitrary factor of the dimension (length)^2, necessary to give the chiral composites the dimension of a scalar field, has been written for convenience in the form $a^2 k_\pi$. The composite action must be $O(4)$ invariant, so that with the inclusion of a linear breaking term it must have the form

$$S_C = \left[\frac{1}{2}(\vec{\pi}, C\vec{\pi})_c + \frac{1}{2}(\sigma, C\sigma) - \frac{1}{a^2}\left(\sqrt{\Omega}m, \sigma\right)\right], \tag{.8}$$

with $\vec{\pi}\cdot\vec{\pi} = \pi_1^2 + \pi_2^2 + \pi_3^2$. The factor $\sqrt{\Omega}$ (we remind that Ω is the number of quark components), has been introduced for later convenience. We choose [6] the wave operator

$$C = a^{-2}\frac{\rho^4}{a^2\Box - \rho^2}, \tag{.9}$$

where $\Box$ is the laplacian on the lattice. The irrelevance of S_C requires that the parameter ρ be independent on the lattice spacing and that k_π do not diverge in the continuum limit. These constraints, with the dependence on a assumed below for the breaking parameter m, ensures also the irrelevance of the chiral symmetry breaking term.

We will now sketch the derivation of a perturbative expansion for the partition function of QCD. We start by using the Stratonovich-Hubbard transformation [7] to replace the chiral composites by auxiliary fields

$$\exp[-S_C] = \left[\frac{d\vec{\chi}}{\sqrt{2\pi}}\right]\left[\frac{d\phi}{\sqrt{2\pi}}\right]\exp[-S_\chi]\,\triangle\,\exp\left(\overline{\lambda}, \frac{1}{a}D\lambda\right), \tag{.10}$$

where

$$\begin{aligned} S_\chi &= -\frac{1}{2}\rho^4\left[(\vec{\chi}, (a^4C)^{-1}\vec{\chi}) + (\phi, (a^4C)^{-1}\phi)\right] \\ &\quad -\frac{\Omega}{2}\sum_x \ln\left\{a^2k_\pi^2\left[\left(\sqrt{\Omega}m + \rho^2\phi(x)\right)^2 + \rho^4\vec{\chi}(x)^2\right]\right\}, \\ \triangle &= \prod_x (\det D(x))^{-1}, \\ D &= ak_\pi\left[\rho^2\phi + \sqrt{\Omega}m + i\rho^2\gamma_5\vec{\tau}\cdot\vec{\chi}\right]. \end{aligned} \tag{.11}$$

The partition function can then be written

$$\begin{aligned} Z = &\int [dU]\exp[-S_{YM}]\left[\frac{d\vec{\chi}}{\sqrt{2\pi}}\right]\left[\frac{d\phi}{\sqrt{2\pi}}\right]\exp[-S_\chi] \\ &\triangle\int[d\bar{\lambda}d\lambda]\exp\left(\overline{\lambda}, (\frac{1}{a}D + Q)\lambda\right). \end{aligned} \tag{.12}$$

The minimum of S_χ occurs at $\vec{\chi} = 0$, but $\phi \neq 0$, implying a breaking of the chiral symmetry. Since Ω is a rather large number we can apply the saddle-point method and perform an expansion in inverse powers of this parameter [6]. The validity of this expansion is subject [8] to the condition $\rho \sim 1$ mentioned in the introduction. S_χ contains a quadratic part plus interactions which vanish in the continuum limit (but have finite quantum effects). The masses associated to the fields ϕ, χ and the mass of the quarks as functions of ϕ are

$$\begin{aligned} m_+^2 &= \frac{\rho^2}{a^2}\left\{1 + \left(\frac{1}{\sqrt{\Omega}}\rho a\phi + \frac{1}{\rho}am\right)^{-2}\right\} \\ m_-^2 &= \frac{\rho^2}{a^2}\left\{1 - \left(\frac{1}{\sqrt{\Omega}}\rho a\phi + \frac{1}{\rho}am\right)^{-2}\right\}, \\ M_Q &= k_\pi(\rho^2\phi + \sqrt{\Omega}m) + m_Q. \end{aligned} \tag{.13}$$

The minimum of S_χ wr to ϕ occurs at

$$\phi = \overline{\phi} \sim \frac{\sqrt{\Omega}}{a\,\rho}(1 - \xi), \tag{.14}$$

where

$$\xi = \frac{1}{2\rho}am. \tag{.15}$$

The mass of the pion coincides with m_-. To have $m_- = m_\pi$ we must assume the breaking parameter

$$m = \frac{1}{\rho}a\, m_\pi^2. \tag{.16}$$

The mass of the σ, which is equal to m_+, diverges in the continuum limit.

The quarks have a mass $M_Q \sim \sqrt{\Omega}\rho k_\pi/a$, which seems also to diverge in the continuum limit. Whether this is actually true depends on the behaviuor of k_π with the lattice spacing. An exemple of the possible need of a scaling of k_π with a is provided by the expression of the pion-nucleon coupling constant [8]. But irrespective of this dependence the quarks are nonpropagating at the perturbative level. Since their mass is of order $\sqrt{\Omega}$, the saddle point expansion results to be a hopping expansion as far as the quarks are concerned, and as a consequence, the quark correlator connecting two sites which are n lattice spacings apart from one another gets the first nonvanishing contribution at an order of the expansion not smaller than n: The quarks are never produced to any finite order. In this sense we can say that they are confined at the perturbative level.

In connection with the nature of the phase transition, it is essential to observe that the above result is solely due to the spontaneous chiral symmetry breaking, in whose absence there would be no saddle point expansion, and does not depend on the explicit breaking term. Since the quark confinement is due to the spontaneous chiral symmetry breaking, the chiral symmetry restoration coincides with the quark deconfinement: They are one and the same phase transition.

We also note that since in the confining phase the quarks have no poles whatsoever, it seems that we do not have to worry about the spurious ones. In any case we can safely assume the quark Wilson parameter r_q of order $1/\sqrt{\Omega}$, so that we can neglect the quark Wilson term to leading order in our expansion.

Introducing the fluctuation of the ϕ-field

$$\theta = \phi - \overline{\phi}, \tag{.17}$$

we arrive at the desired expansion of the partition function of QCD in terms of the auxiliary fields. Since only terms with an even power of S_Q can contribute, we have

$$Z = \int [dU] \exp[-S_{YM}] \left[\frac{d\vec{\chi}}{\sqrt{2\pi}}\right] \left[\frac{d\phi}{\sqrt{2\pi}}\right] \exp[-S_\chi] \triangle \int [d\bar{\lambda} d\lambda] \sum_{r=0}^{\infty} \frac{1}{(2r)!} (S_Q)^{2r} \exp\left(\overline{\lambda}, \frac{1}{a} D\lambda\right) = \sum_{r=0}^{\infty} Z_r. \tag{.18}$$

This is an expansion in inverse powers of k_π, because obviously $Z_r \sim k_\pi^{-2r}$. Each term Z_r is a function of Ω which can be given a series in Ω^{-1}. The study of the pion-pion interaction determines the value of ρ and the scaling of k_π with the lattice spacing [9]. Note that we do not need to treat the gluon field perturbatively: If and where this can possibly be done remains here an open question.

Finally we evaluate the critical temperature in a one loop approximation, by the condition that the effective potential be a minimum for vanishing pion mass, namely for

$$\phi = \overline{\phi} - \frac{\sqrt{\Omega}}{2\rho^3} a\, m_\pi^2. \tag{.19}$$

For this value of ϕ the mass of the σ remains divergent in the continuum limit: the chiral symmetry restoration-quark deconfinement is a first order phase transition.

It is convenient to introduce the field φ

$$\overline{\phi}(T) = \overline{\phi} + a\varphi, \tag{.20}$$

because the pion mass vanishes for a finite value of it

$$\overline{\varphi} = -\frac{\sqrt{\Omega}}{2\rho^3} m_\pi^2. \tag{.21}$$

Then, since φ has dimension 2, we can confine ourselves to approximations quadratic in this field. Let us separate the effective potential in its various parts

$$V = V_0 + V_1 + V_T. \tag{.22}$$

V_0 is the classical potential

$$\begin{aligned} V_0 &= \frac{1}{2a^2}\rho^2\phi^2 - \Omega\frac{1}{a^4} \ln\left(\sqrt{\Omega} am + \rho^2 a\phi\right) \\ &\sim \text{const} + \rho^2\varphi^2. \end{aligned} \tag{.23}$$

V_1 is the one loop contribution at zero temperature. Its quadratic approximation, under the usual normalization conditions

$$\frac{\partial}{\partial\varphi} V_1 = \frac{\partial^2}{\partial\varphi^2} V_1 = 0, \tag{.24}$$

must vanish identically. This has been checked by an explicit evaluation of the counterms δm and $\delta\rho$ necessary to cancel out the one loop divergences. The calculation has been performed for simplicity by considering the action S_χ in a continuous euclidean space, whith a cutoff $1/a$. Then the contribution coming from the counterterms is

$$\begin{aligned} \overline{V}_1 = &\frac{\sqrt{\Omega}}{b^2a^2}\left(a\delta m + \frac{\delta\rho^2}{\rho b}\right)\varphi \\ &+ \left[\left(\frac{1}{2} + \frac{1}{b^2}\right)\delta\rho^2 - \frac{\rho}{b^3}\left(a\delta m + \frac{\delta\rho^2}{\rho b}\right)\right]\varphi^2. \end{aligned} \tag{.25}$$

The total zero temperature one loop potential is

$$V_1 = \overline{V}_1 + F(m_+^2) + 3F(m_-^2), \tag{.26}$$

where

$$F(m^2) = \frac{1}{16(2\pi)^2}\left[\frac{1}{2}m^4 + 4a^{-2}m^2 - m^4 \ln\frac{4}{a^2m^2}\right]. \tag{.27}$$

The approximations to $m_\pm^2$, quadratic in the field φ, are

$$\begin{aligned} m_+^2 &= \frac{2\rho^2}{a^2}\left(1 - \xi - \frac{\rho}{\sqrt{\Omega}}a^2\varphi - \frac{3\rho^2}{2\Omega}a^4\varphi^2\right) \\ m_-^2 &= \frac{2\rho^2}{a^2}\left(\xi + \frac{\rho}{\sqrt{\Omega}}a^2\varphi + \frac{3\rho^2}{2\Omega}a^4\varphi^2\right). \end{aligned} \tag{.28}$$

¿From the cancellation of the divergent terms we get

$$\begin{aligned} \delta\rho^2 &= \frac{1}{(2\pi)^2}\frac{\rho^6}{\Omega}\left[-\frac{3}{2} + \frac{1}{3\rho^2} - \frac{1}{3}\ln\frac{\rho^2}{2} - \frac{1}{2}\ln\frac{\rho^2\xi}{2}\right] \\ \delta m &= \frac{1}{a}\frac{1}{(2\pi)^2}\frac{\rho^5}{\Omega}\left[2 - \frac{4}{3\rho^2} + \frac{5}{6}\ln\frac{\rho^2}{2} + \frac{1}{2}\ln\frac{\rho^2\xi}{2}\right], \end{aligned} \tag{.29}$$

which make V_1 identically zero.

Finally V_T is the temperature dependent one loop part

$$V_T = V^+(T) + 3V^-(T) - V^+(0) + 3V^-(0), \tag{.30}$$

where

$$\begin{aligned} V^\pm(T) = &\frac{T}{2(2\pi)^3}\int_{-\frac{\pi}{a}}^{+\frac{\pi}{a}} d^3p \\ &\sum_n \ln\left(a^{-2}\sin^2\left(\frac{2\pi}{N_t}n\right) + p^2 + m_\pm^2\right). \end{aligned} \tag{.31}$$

$V^{\pm}$ has been evaluated in [1] for $T \gg m$. Since we do not need its full expression, but only its derivative, we can avoid such limitation. Now

$$\frac{\partial}{\partial m_{\pm}^2} V^{\pm} = \frac{T}{2(2\pi)^3} \int_{-\frac{\pi}{a}}^{+\frac{\pi}{a}} d^3p \sum_n \left(a^{-2} \sin^2 \left(\frac{2\pi}{N_t} n \right) + p^2 + m_{\pm}^2 \right)^{-1}. \quad (.32)$$

Taking the limit $a \to 0$ at finite T we get

$$\frac{\partial}{\partial m_{\pm}^2} V^{\pm} = \frac{T}{2(2\pi)^3} \int_{-\infty}^{+\infty} d^3p \sum_n \left((2\pi n T)^2 + p^2 + m_{\pm}^2 \right)^{-1}. \quad (.33)$$

We see that the σ does not contribute because of its divergent mass, while the pion contribution for vanishing pion mass is

$$\frac{\partial}{\partial m_{-}^2} \left[V^{-}(T) - V^{-}(0) \right] = \frac{1}{2(2\pi)^3} \int_{-\infty}^{+\infty} d^3p \frac{1}{p} \left[\exp(p^2/T) - 1 \right]^{-1} = \frac{1}{24} T^2. \quad (.34)$$

In conclusion

$$\frac{\partial}{\partial \varphi} V = 2\rho^2 \varphi + \frac{\rho^3}{4\sqrt{\Omega}} T^2. \quad (.35)$$

¿From the condition that the effective potential be a minimum for vanishing pion mass, namely for $\varphi = \overline{\varphi}$ we get the critical temperature

$$T = \frac{2\sqrt{\Omega}}{\rho^2} \, m_{\pi}. \quad (.36)$$

To have an estimate of the accuracy of this result one should evaluate the two loop contribution to the effective potential taking into account the quark contribution in the expansion in k_{π}^{-1}. Another possibility is to use the formulation (.12) in a numerical simulation. This might have the advantage of an easier evaluation the quark determinant and of the guidance of the above result.

We conclude by observing that our approach appears promising also in the study of the phase transition at high baryon density [10] because, unlike the standard way, where the chemical potential acts on the quarks to allow the integration over the latters to be performed analytically, we can couple it directly to the nucleons, with a significant simplification.

[1] D. A. Kirzhnits, JEPT Lett. 15 (1972) 529; D. A. Kirzhnits and A. D. Linde, Phys. Lett. 42B (1972) 471; S. Weinberg, Phys. Rev. D9 (1974) 3320; L. Dolan and R. Jackiw, Phys. Rev. D9 (1974) 3357

[2] F. Kars, Simulating the quark-gluon plasma on the lattice, Advanced Series on Directions in High Energy Physics- Vol.6 (1990) 61,"Quark Gluon Plasma" (Edt. R.C. Hwa), Singapore 1990, Word Scientific; K. Kanaya, An introduction to finite temperature QCD on the lattice, Prog. Theor. Phys. Suppl. 131 (1998) 73

[3] F. Kars, Hep-Lat/9903031 v2 17 Mar 1999

[4] F. Palumbo, Phys. Lett. B328 (1994) 79; G. De Franceschi and F. Palumbo, Mod. Phys. Lett. A11 (1995) 901.

[5] G. DeFranceschi and F. Palumbo, hep-th/9810218; Phys. Rev. D, submitted to

[6] S. Caracciolo and F. Palumbo, Nucl. Phys. B512 (1998) 505

[7] R. L. Stratonovitch, Dokl. Acad. Nauk. S.S.S.R. 115 (1957) 1097 [Sov. Phys. Dokl. 2 (1958) 416]; J. Hubbard, Phys. Rev. Lett. 3 (1959) 77

[8] F. Palumbo, Phys. Rev.D, in press

[9] S. Caracciolo, F. Palumbo and R.Scimia, in preparation

[10] QCD at finite baryon density, Bielefeld 1998, Nucl. Phys. A 642 (1998)